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The influence of a slowly varying axial magnetic field on the stability of a gravitating cylinder

BY P. K. BHAT

*Department of Applied Mathematics, Indian Institute of Science, Bangalore-12, India**

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The effect of a slowly varying axial magnetic field on the stability of a gravitating cylinder is considered. In the case of axisymmetric disturbances it is found that whenever the system is unstable the effect of this small inhomogeneity of the field is to add to the instability of the system. Further, an increase in the magnitude of of the magnetic field decreases the wave number thereby stabilizing the long wave length disturbances. In the case of two dimensional disturbances it is found that the stability of the system is not influenced by the functional form $f(r)$, r being the radial co-ordinate, of the axial magnetic field within the cylinder, in the absence of surface curreuts.

INTRODUCTION

The problem of the stability of a gravitating cylinder in the presence of a magnetic field has been the subject of extensive study due to its importance in many astrophysical phenomena Chandrasekhar & Fermi (1953) have discussed the problem subject to the influence of a uniform axial magnetic field. Auluck & Kothari (1957) have investigated the problem in the absence of surface currents for both poloidal and toroidal magnetic fields using the energy method. They find that the effect of the magnetic field is to increase the stability of the system. Chakraborty & Bhatnagar (1960) have investigated the effect of uniform volume current and surface charge on the stability of a self gravitating liquid column using the method of normal modes They find that the system is unstable against axisymmetric disturbances of all wave lengths lying in some definite interval.

In part A of this paper we have studied the stability of an infinitely long incompressible and infinitely conducting gravitating cylinder in the presence of a twisted magnetic field. We have assumed that the axial magnetic field prevailing inside the cylinder varies slowly in space. Applying the method of normal modes we have studied the effect of this small inhomogeneity (in the axial magnetic field) on the longitudinal stability of the system. In part B we have discussed the stability of the system against azimuthal disturbances. In this case we have made no assumption on the nature of the axial magnetic field prevailing inside the cylinder.

*Present address : School of Mathematics, Military College of Engineering Dapodi, Poona–31.

PART–A

Initial state

We consider the longitudinal stability of an infinite cylinder which is infinitely conducting and self gravitating under the influence of the following magnetic field configurations

$$\vec{H}_0^{(i)} = [0,\ Br,\ H_0 - \tfrac{1}{2}H_1 r^2] \qquad (r \leqslant R) \qquad \ldots \quad (2.1)$$

$$\vec{H}_0^{(o)} = \left[0,\ \frac{BR^2}{r},\ H_0 - \tfrac{1}{2}H_1 R^2\right] \quad (r \geqslant R) \qquad \ldots \quad (2.2)$$

where the superscripts (i) and (o) denote the inside and the outside of the cylinder, R the radius and B, H_0 and H_1 are constants. Taking H_0 as the standard magnetic field, R as the characteristic length and μH_0^2 as the characteristic pressure, (2.1) and (2.2) can be written as

$$\vec{H}_0^{(i)} = [0,\ \lambda r,\ 1 - \alpha r^2] \qquad (r \leqslant 1) \qquad \ldots \quad (2.3)$$

$$\vec{H}_0^{(i)} = [0,\ \lambda / r,\ 1 - \alpha], \quad (r \geqslant 1) \qquad \ldots \quad (2.4)$$

where $\lambda = BR/H_0$ and $\alpha = \frac{1}{2}\ H_1 R^2/H_0$. The dimensionless pressure inside the cylinder is given by

$$P_0^{(i)} = \alpha(r^2-1) + \alpha^2/2.(1-r^4) + \lambda^2(1-r^2) + \beta/2.(1-r^2), \quad (r \leqslant 1) \quad \ldots \quad (2.4')$$

where $\beta = 2\pi G\rho R^2/\mu H_0^2$ and the dimensionless gravitational potentials inside and outside the cylinder are respectively given by

$$\phi_0^{(i)} = \tfrac{1}{2}(1-r^2) \qquad (r \leqslant 1) \qquad \ldots \quad (2.5)$$

and

$$\phi_0^{(0)} = -\ln r. \qquad (r \geqslant 1) \qquad \ldots \quad (2.6)$$

Linearised Equations

The equations governing the small perturbation in the physical quantities inside the cylcinder are

$$\frac{\partial \tilde{v}_r}{\partial t} = -\frac{\partial \tilde{p}}{\partial r} + (1-\alpha r^2)\left(\frac{\partial \tilde{H}_r}{\partial z} - \frac{\partial \tilde{H}_z}{\partial r}\right) - \lambda \frac{\partial}{\partial r}(r\tilde{H}_\theta)$$

$$+ 2\alpha r\tilde{H}_z - 2\lambda\tilde{H}_\theta + \beta\,\frac{\partial \tilde{\phi}}{\partial r}, \qquad \ldots \quad (3.1)$$

$$\frac{\partial \tilde{v}_\theta}{\partial t} = (1-\alpha r^2)\,\frac{\partial \tilde{H}_\theta}{\partial z} + 2\lambda\tilde{H}_r, \qquad \ldots \quad (3.2)$$

$$\frac{\partial \tilde{v}_z}{\partial t} = -\frac{\partial \tilde{p}}{\partial z} - \lambda r\,\frac{\partial \tilde{H}_\theta}{\partial z} - 2\alpha r\tilde{H}_r + \beta\,\frac{\partial \tilde{\phi}}{\partial z}, \qquad \ldots \quad (3.3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{v}_r)+\frac{\partial \tilde{v}_z}{\partial z}=0, \qquad \ldots \quad (3.4)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{H}_r)+\frac{\partial \tilde{H}_z}{\partial z}=0, \qquad \ldots \quad (3.5)$$

$$\frac{\partial \tilde{H}_r}{\partial t}=(1-\alpha r^2)\frac{\partial \tilde{v}_r}{\partial z}, \qquad \ldots \quad (3.6)$$

$$\frac{\partial \tilde{H}_\theta}{\partial t}=(1-\alpha r^2)\frac{\partial \tilde{v}_\theta}{\partial z}, \qquad \ldots \quad (3.7)$$

$$\frac{\partial \tilde{H}_z}{\partial t}=-1/r\frac{\partial}{\partial r}\{r(1-\alpha r^2)\tilde{v}_r\}, \qquad \ldots \quad (3.8)$$

$$\frac{\partial^2 \varphi}{\partial r^2}+1/r\frac{\partial \varphi}{\partial r}+\frac{\partial^2 \varphi}{\partial z^2}=0, \qquad \ldots \quad (3.8a)$$

$$\dot{E}+v\times H_0=0, \qquad \ldots \quad (3.8b)$$

where $\vec{v}=(\tilde{v}_r, \tilde{v}_\theta, \tilde{v}_z)$, $H'=(\tilde{H}_r, \tilde{H}_\theta, \tilde{H}_z)$, $\tilde{p}$, $\tilde{\phi}$ and E' are perturbations in velocity, magnetic field, pressure, gravitational potential and the electric field respectively. It may be noted that for characteristic velocity we have taken the Alfvén velocity.

We assume that the disturbances are of the type $x=\hat{x}(r)\exp(i\omega t+ikz)$, where ω is the frequency and k is the wave number, so that the equations (3.1)—(3.8b) reduce to the following form

$$i\omega\hat{v}_r=-\frac{d\hat{p}}{dr}+(1-\alpha r^2)\left(ik\hat{H}_r-\frac{d\hat{H}_z}{dr}\right)-\lambda\frac{d}{dr}(r\hat{H}_\theta)$$

$$+2\alpha r\hat{H}_z-2\lambda\hat{H}_\theta+\beta\frac{d\hat{\phi}}{dr}, \qquad (3.9)$$

$$i\omega\hat{v}_\theta=ik(1-\alpha r^2)\hat{H}_\theta+2\lambda\hat{H}_r, \qquad (3.10)$$

$$i\omega\hat{v}_z=-ik\hat{p}-ik\lambda r\hat{H}_\theta-2\alpha r\hat{H}_r+ik\beta\hat{\phi}, \qquad (3.11)$$

$$1/r\,.\,d/dr\,.\,(r\hat{v}_r)+ik\hat{v}_z=0, \qquad (3.12)$$

$$i\omega\hat{H}_r=ik(1-\alpha r^2)\hat{v}_r, \qquad (3.13)$$

$$i\omega\hat{H}_\theta=ik(1-\alpha r^2)\hat{v}_\theta, \qquad (3.14)$$

$$i\omega\hat{H}_z=-1/r\,.\,d/dr\,.\,\{r(1-\alpha r^2)\hat{v}_r\}, \qquad (3.15)$$

$$\frac{d^2\hat{\phi}}{dr^2}+\frac{1}{r}\,\frac{d\hat{\phi}}{dr}-k^2\phi=0, \qquad \text{... (3.15a)}$$

$$\vec{\hat{E}}+\vec{\hat{v}}\times\vec{H}_0=0. \qquad \text{... (3.15b)}$$

In passing we note that equations (3.6) and (3.8) are quivalent in view of equation (3.5) so we shall drop equation (3.5).

From equations (3.9)—(3.15) we have

$$r^2\{\omega^2-k^2(1-\alpha r^2)^2\}\frac{d^2\hat{v}_r}{dr^2}+\{\omega^2 r-k^2r(1-\alpha r^2)(1-5\alpha r^2)\}\,\frac{d\hat{v}_r}{dr}$$
$$+\{k^4r^2(1-\alpha r^2)^2-\omega^2-\omega^2k^2r^2+k^2(1-\alpha r^2)(1+3\alpha r^2)$$
$$+\frac{4\lambda^2k^4r^2(1-\alpha r^2)^2}{\omega^2-k^2(1-\alpha r^2}\Big\}\,\hat{v}_r=0 \qquad \text{... (3.16)}$$

We shall assume that α is very small so that its squares and higher powers can be neglected. Expanding the physical quantities involved in powers of α we have

$$\begin{aligned}
\acute{v} &= v_0+\alpha\,\acute{v}_1+0(\alpha^2),\\
\hat{p} &= \hat{p}_0+\alpha\hat{p}_1+0(\alpha^2),\\
\acute{H} &= \acute{H}_0+\alpha\acute{H}_1+0(\alpha^2),\\
\hat{\phi} &= \hat{\phi}_0+\alpha\hat{\phi}_1+0(\alpha^2),\\
\acute{E} &= \acute{E}_0+\alpha\acute{E}_1+0(\alpha^2),\\
\omega &= \omega_0+\alpha\omega_1+0(\alpha^2).
\end{aligned} \qquad \text{... (3 17)}$$

Substituting (3.17) in (3.16) and separating the various order terms we have

$$\frac{d^2\hat{v}_{r0}}{dr^2}+\frac{1}{r}\,\frac{d\hat{v}_{r0}}{dr}-\left\{k^2-\frac{4\lambda^2k^4}{(\omega_0{}^2-k^2)^2}+\frac{1}{r^2}\right\}\hat{v}_{r0}=0. \qquad \text{... (3.18)}$$

$$\frac{d^2v_{r1}}{dr^2}+\frac{1}{r}\,\frac{d\hat{v}_{r1}}{dr}-\left\{k^2-\frac{4\lambda^2k^4}{(\omega_0{}^2-k^2)^2}+\frac{1}{r^2}\right\}\hat{v}_{r1}$$
$$=\frac{4ik^3r}{(\omega_0{}^2-k^2)}\,\hat{v}_{z0}+\frac{8\lambda^2k^4}{(\omega_0{}^2-k^2)^3}\{\omega_0{}^2r^2+2\omega_0\omega_1+k^2r^2\}\hat{v}_{r0} \qquad \text{... (3.19)}$$

Zeroeth order solutions

$$\hat{v}_{r0}=AI_1(lr) \qquad \text{... (4.1)}$$

where A is an arbitrary constant of integration, I_1 the modified Bessel function of order unity and $l^2=k^2-\dfrac{4\lambda^2k^2}{(\omega_0^2-k^2)^2}\cdot$

Now from (3.9)–(3.15), (3.17) and (4.1) we have

$$\left.\begin{aligned}
\hat{v}_{\theta 0} &= -\frac{2ik\lambda A}{(\omega_0^2-k^2)}\, I_1(lr),\\
\hat{v}_{z0} &= \frac{iAl}{k}\, I_0(lr),\\
\hat{H}_{r0} &= \frac{kA}{\omega_0}\, I_1(lr),\\
\hat{H}_{\theta 0} &= -\frac{2i\lambda k^2 A}{\omega_0(\omega_0^2-k^2)}\, I_1(lr),\\
\hat{H}_{z0} &= \frac{iAl}{\omega_0}\, I_0(lr),\\
\hat{p}_0 &= \beta\hat{\phi}_0+\frac{2i\lambda^2 k^2 Ar}{\omega_0(\omega_0^2-k^2)}\, I_1(lr)\\
&\quad -\frac{iA\omega_0 l}{k^2}\, I_0(lr).
\end{aligned}\right\} \tag{4.2}$$

Again from (3.15a) and (3.17) we get

$$\frac{d^2\hat{\phi}_0}{dr^2}+\frac{1}{r}\frac{d\hat{\phi}_0}{dr}-k^2\hat{\phi}_0 = 0, \qquad \ldots \quad (4.3)$$

and

$$\frac{d^2\hat{\phi}_1}{dr^2}+\frac{1}{r}\frac{d\hat{\phi}_1}{dr}-k^2\hat{\phi}_1 = 0. \qquad \ldots \quad (4.3a)$$

Solving (4.3) we get

$$\hat{\phi}_0 = B_1 I_0(kr), \qquad \ldots \quad (4.4)$$

where B_1 is an arbitrary constant of integration. Similarly from (3.15b), (3.17) and (4.2) we have

$$\left.\begin{aligned}
\hat{E}_{r0} &= \frac{iA\lambda lr}{k}\, I_0(lr)+\frac{2iA\lambda K}{(\omega_0^2-k^2)}\, I_1(lr),\\
\hat{E}_{\theta 0} &= AI_1(lr),\\
\hat{E}_{z0} &= -A\lambda r I_1(lr).
\end{aligned}\right\} \qquad \ldots \quad (4.5)$$

Solution of first order equations

From (3.19) and (4.2) we have

$$\frac{d^2\hat{v}_{r1}}{dr^2}+\frac{1}{r}\frac{d\hat{v}_{r1}}{dr}-\left(l^2+\frac{1}{r^2}\right)\hat{v}_{r1}=M_1(lr)\,I_0(lr)$$

$$+M_2l^2r^2I_1(lr)+M_3I_1(lr), \qquad \ldots \quad (5.1)$$

where

$$\left.\begin{aligned} M_1 &= -\frac{4Ak^2}{(\omega_0{}^2-k^2)}, \\ M_2 &= \frac{8A\lambda^2k^4(\omega_0{}^2+k^2)}{(\omega_0{}^2-k^2)^3}, \\ \text{and} \quad M_3 &= \frac{16A\lambda^2k^4\omega_0\omega_1}{(\omega_0{}^2-k^2)^3}. \end{aligned}\right\} \qquad (5.2)$$

Equation (5.1) yields (Mclachlan 1934).

$$v_{r1}=M_1'\frac{l^2r^2}{4}I_1(lr)+M_2'\frac{l^3r^3}{6}I_2(lr)+M_3'\frac{lr}{2}I_2(lr), \qquad \ldots \quad (5.3)$$

where $\quad M_1'=\frac{M_1}{l^2},\quad M_2'=\frac{M_2}{l^4}$ and $M_3'=\frac{M_3}{l^2}$

Thus to this order of approximation, we get

$$\hat{v}_r=AI_1(lr)+\alpha\left\{M_1'\frac{l^2r^2}{4}I_1(lr)+M_2'\frac{l^3r^3}{6}I_2(lr)+M_3'\frac{lr}{2}I_2(lr)\right\}. \qquad \ldots \quad (5.4)$$

Similarly, we can find the other physical variables to this order of approximation.

Solutions in vacuum

$$\text{Curl}\ \vec{\tilde{H}}=0. \qquad \ldots \quad (6.1)$$

$$\text{Div}\ \vec{\tilde{H}}=0, \qquad \ldots \quad (6.2)$$

$$\text{Div}\ \vec{\tilde{E}}=0, \qquad \ldots \quad (6.3)$$

$$\text{Curl}\ \vec{\tilde{E}}=-\frac{\partial\vec{\tilde{H}}}{\partial t}, \qquad \ldots \quad (6.3)$$

and

$$\Delta^2\tilde{\psi}=0, \qquad \ldots \quad (6.4)$$

where $\vec{\tilde{H}}$, $\vec{\tilde{E}}$ and $\tilde{\psi}$ are the disturbances in the magnetic field, the electric field and the gravitational potential, respectively.

Solving (6.1)–(6.4), we get

$$\hat{\vec{H}} = [-Ckk_1(kr),\, 0,\, iCkk_0(kr)]$$

$$\hat{\vec{E}} = [iDk_1(kr),\, -\omega Ck_1(kr), Dk_0(kr)], \tag{6.5}$$

and $$\hat{\psi} = Ek_0kr)$$

where C, D and E are arbitrary constants of integration.

Boundary conditions

The equation of the disturbed boundary is $r = 1+(\tilde{\delta r})\exp(i\omega t+ikz)$ and the linearised dynamical and electromagnetic boundary conditions are

$$\vec{n}.[\tilde{\vec{H}}]+\tilde{\vec{n}}.[\vec{H}] = 0.$$

$$\vec{n}\times[\tilde{\vec{H}}]+\tilde{\vec{n}}\times[\vec{H}] = \tilde{\vec{J}}^*$$

$$\vec{n}.[\tilde{\vec{E}}]+\tilde{\vec{n}}.[\vec{E}] = \tilde{q}^*,$$

$$\vec{n}\times[\tilde{\vec{E}}]+\tilde{\vec{n}}\times[\vec{E}] = \tilde{u}[\vec{H}]+u[\tilde{\vec{H}}],$$

$$\vec{n}[\tilde{p}]+\tilde{\vec{n}}[p] = \vec{J}^*\times\tilde{\tilde{\vec{H}}}+\tilde{\vec{J}}^*\times\bar{\vec{H}}+q^*\tilde{\vec{E}}+\tilde{q}^*\vec{E},$$

$$\tilde{\phi} = \tilde{\psi}, \qquad \frac{\partial\tilde{\phi}}{\partial r} = \frac{\partial\tilde{\psi}}{\partial r}+2\tilde{\delta r},$$

and $$n.\,v+n.\,v = u, \qquad \ldots \quad (7.1)$$

where [ ] denotes the jump in the value of a physical quantity in going from the inside of the cylinder to the outside vaccum. $\vec{n}$, $\vec{j}^*$, q^* and u denote the unit outward normal to the unperturbed boundary, the surface current density before the disturbance, the surface charge density before the disturbance and the velocity of the unperturbed boundary respectively. $\tilde{\vec{n}}$, $\tilde{\vec{j}}^*$, $\tilde{q}^*$ and $\tilde{u}$ denote the perturbations in the unit outward normal, the surface current density, the surface charge density and the velocity of the boundary respectively and $\bar{\vec{H}}$, denotes the mean of the magnetic field strength just inside and just outside the cylinder.

The perturbation in the unit normal to the boundary is given by

$$\tilde{\vec{n}} = [0,\, 0,\, -ik\tilde{\delta r}\exp(i\omega t+ikz)]. \tag{7.2}$$

After applying the set of boundary conditions (7.1) at the disturbed boundary $r = 1+(\widetilde{\delta r})\exp(i\omega t+ikz)$, we get the following dispersion relations

$$k^2[\beta k_1(k)I_1(l)-2\beta I_0(k)k_0(k)k_1(k)I_1(l)+ +\{kk_0(k)I_1(l)+lI_0(l)k_1(l)\}]-\omega_0^2 lI_0(l)k_1(k) = 0. \quad \ldots \quad (7.3)$$

$$\omega_1 = \left[2I_1(l)-\frac{2kk_0(k)I_1(l)}{k_1(k)}-2lI_0(l)+M_2''\Omega_1\right]\left[\frac{2\omega_0 lI_0(l)}{k^2}-M_3'''\Omega_2\right]^{-1} \ldots \quad (7.4)$$

where

$$\left.\begin{aligned} M_2'' &= \frac{8\lambda^2k^4(\omega_0^2+k^2)}{(\omega_0^2-k^2)l^4}, \\ M_3''' &= \frac{16\lambda^2k^4\omega_0}{(\omega_0^2-k^2)^3l^2} \\ \Omega_1 &= \frac{l^4(\omega_0^2-k^2)}{6k^2}\cdot\frac{I_0(l)\,I_2(l)}{I_1(l)}-\frac{l^4(\omega_0^2-k^2)}{12k^2}\{3I_1(l)-I_3(l)\}, \\ \Omega_2 &= \frac{l^2(\omega_0^2-k^2)}{2k^2}\cdot\frac{I_0(l)I_2(l)}{I_1(l)}-\frac{l^2(\omega_0^2-k^2)}{2k^2}I_1(l). \end{aligned}\right\} \quad \ldots \quad (7.5)$$

Discussion of Results

We have discussed the following cases

(i) $\lambda = 0$:

In this case the azimuthal component of the initial magnetic field is zero. The dispersion relations (7.3) and (7.4) have the following forms respectively,

$$\frac{k}{I_0(k)\,k_1(k)}\,[\beta k_1(k)\,I_1(k)-2\beta I_0(k)k_0(k)I_1(k)k_1(k)+1], \quad \ldots \quad (8.1)$$

$$\omega_1 = \pm\frac{k}{I_0(k)k_1(k)}\,\{I_1(k)k_1(k)-1\}\div \left[\frac{k}{I_0(k)k_1(k)}\,\{\beta k_1(k)I_1(k)-2\beta I_0(k)k_0(k)I_1(k)k_1(k)+1\}\right]^{\frac{1}{2}} \quad \ldots \quad (8.2)$$

From (8.1) it follows that the system is stable for all values of $k \geqslant 1.0668$ whatever positive value β may take. In case $k < 1.0668$ the system is stable if

$$\beta < \left\{\frac{1}{I_1(k)k_1(k)\{2I_0(k)k_0(k)-1\}}\right\},$$

and unstable if

$$\beta > \left\{\frac{1}{I_1(k)k_1(k)\{2I_0(k)k_0(k)-1\}}\right\}$$

Now (8.2) implies that the effect of the inhomogeneity is to increase the frequency of stable oscillations and if the system is unstable its effect is to increase the growth of instability by a multiple of ω_1.

(ii) $\lambda = 0,\ \beta = 0$

When the azimuthal magnetic field and the gravitational force are absent, the dispersion relations (7.3) and (7.4) take the forms

$$\omega_0^2 = \left\{\frac{k}{I_0(k)\,k_1(k)}\right\}, \qquad \ldots \quad (8.3)$$

and

$$\omega_1 = \pm \frac{k}{I_0(k)k_1(k)}\{I_1(k)k_1(k)-1\} \div \left\{\frac{k}{I_0(k)k_1(k)}\right\}. \qquad \ldots \quad (8.4)$$

From (8.3) it follows that the system is stable for all real positive values of the wave number k and (8.4) shows that inhomogeneity adds to stability of the system.

(iii) $\lambda \neq 0,\ \beta \neq 0$

We find that $[\omega_0^2-\{k^2\pm 2\lambda k\}]$, is root of (7.5). This implies that the system is stable for all wave numbers $k > 2\lambda$ and unstable for all wave numbers $k < 2\lambda$ In case $k > 2\lambda$ the effect of the inhomogeneity of the magnetic field is to add to the stability of the system and if $k < 2\lambda$ its effect is to enhance the growth rate of instability by a multiple of

$$\omega_1 = \pm\left[1+\frac{kk_0(k)}{k_1(k)}\right] k^2 \div \{k^2 \pm 2\lambda k\}^{\frac{1}{2}}$$

Thus defining a critical wave number $k^* = \dfrac{2B}{H_0}$ we find that an increase in the magnitude of the magnetic field decreases the wave number thereby stabilizing long wave length disturbances.

(iv) $\lambda \neq 0,\ \beta \neq 0,\ l \neq 0$

We find that for stable modes the frequency of oscillations is increased by the inhomogeneity of the magnetic field. Further, for such modes we find that (i) for fixed values of λ and β as the wave number k increases the frequency of stable oscillations also increases, (ii) for fixed λ and k the frequency increases when β increases, and (iii) for fixed values of β and k the frequency increases as λ increases (tables 1, 2 and 3).

Part B

In this part we have studied the disturbances of the type $X(r)\exp(i\omega t+im\theta)$, where m is the aximuthal wave number.

TABLE 1

	$\beta = 0$ 0000	$\lambda = 0$ 5000	$\beta = 0$ 0000	$\lambda = 0$ 9000	$\beta = 0$ 000	$\lambda = 2.0000$	$\beta = 0.0500$	$\lambda = 0.0500$	$\beta = 0$ 0500	$\lambda = 0.9000$
k	ω_0^2	ω_1	ω_0^2	ω_1	ω_0^2	ω_1	ω_0^2	ω_1	ω_0^2	ω_1
0.1	0.01116	$\pm$0.00121	0.01208	$\pm$0.00232	0 01462	$\pm$0.00434	0 01447	$\pm$0 00325	0.01778	$\pm$0.00511
0.3	0.12938	$\pm$0.00152	0.15901	$\pm$0.00304	0 18174	$\pm$0.00479	0 10045	$\pm$0 00479	0.10876	$\pm$0.00674
0 5	0.36456	$\pm$0.00185	0.44593	$\pm$0.00415	0 66920	$\pm$0 00493	0 36278	$\pm$0.00613	0.44355	$\pm$0 00732
0.7	0.73014	$\pm$0 00217	0.88208	$\pm$0.00477	1.30938	$\pm$0.00533	0 72714	$\pm$0 00679	0 87957	$\pm$0.00773
0.9	1.23167	$\pm$0.00278	1.47011	$\pm$0.00519	2.15448	$\pm$0 00592	1.22966	$\pm$0.00715	1 46848	$\pm$0.00852
1	1 53619	$\pm$0 00314	1.82153	$\pm$0 00713	2.65094	$\pm$0 00789	1.53524	$\pm$0 00812	1.82078	$\pm$0.00922
3	10.93118	$\pm$0.00513	12.57503	$\pm$0.00755	17 36081	$\pm$0.00809	10.93028	$\pm$0.00888	12 57261	$\pm$0.01213
5	29.01519	$\pm$0.00554	32.28881	$\pm$0.00797	41.51284	$\pm$0.00856	29 01483	$\pm$0.00905	32 28772	$\pm$0.01311

TABLE 2

	$\beta = 0.0500$	$\lambda = 2.0000$	$\beta = 0$ 5000	$\lambda = 0$ 5000	$\beta = 0$ 5000	$\lambda = 0$ 9000	$\beta = 0$ 5000	$\lambda = 2$ 0000	$\beta = 2.0000$	$\lambda = 0.5000$
k	ω_0^2	ω_1	ω_0^2	ω_1	ω_0^2	ω_1	ω_0^2	ω_1	ω_0^2	ω_1
.1	0 01464	$\pm$0.00701	0.02074	$\pm$0.02012	0.02294	$\pm$0 02325	0 03094	$\pm$0 03729	0.11000	$\pm$0.06222
.3	0.13163	$\pm$0 00792	0.14478	$\pm$0.02373	0 17193	$\pm$0 04307	0 25179	$\pm$0 04434	0.25108	$\pm$0.06715
.5	0.66704	$\pm$0.00832	0.36136	$\pm$0 03021	0.44228	$\pm$0.05027	0 66585	$\pm$0 05213	0.48381	$\pm$0.07021
.7	1.30721	$\pm$0.00954	0.70294	$\pm$0.03732	0 85856	$\pm$0 05797	1 28836	$\pm$0.06011	0.74246	$\pm$0 08112
.9	2 15315	$\pm$0.01002	1.21208	$\pm$0.04013	1 45424	$\pm$0 06001	2.14129	$\pm$0.06725	1.16126	$\pm$0.08993
1	2.65034	$\pm$0 01052	1.52686	$\pm$0 04773	1.81411	$\pm$0.06888	2 64494	$\pm$0 07002	1.50004	$\pm$0.09003
3	17.34406	$\pm$0.01312	10.92298	$\pm$0.04921	12.55270	$\pm$0 06929	17.29678	$\pm$0 07773	10 90588	$\pm$0 09135
5	41.50875	$\pm$0.01415	29.01178	$\pm$0 05013	32 27862	$\pm$0.07003	41.47407	$\pm$0 08033	29.00401	$\pm$0.09312

TABLE 3

	$\beta = 2\ 0000$	$\lambda = 0\ 9000$	$\beta = 2.0000$	$\lambda = 2.0000$
k	ω_0^2	ω_1	ω_0^2	ω_1
.1	0.04952	$\pm$.07272	0.05327	$\pm$.09099
.3	0.26404	$\pm$.08013	0 32157	$\pm$ 10155
.5	0.54211	$\pm$.08888	0.74436	$\pm$.17123
.7	0.89223	$\pm$ .09227	1.31802	$\pm$.19773
.9	1 41130	$\pm$.10012	2.10374	$\pm$.21023
1	1.41130	$\pm$.13235	2.62731	$\pm$.26155
3	12 50422	$\pm$ 20007	17.14323	$\pm$.29002
5	32.25507	$\pm$.23135	41.38087	$\pm$ 31013

Starting with the magnetic field configuration given by (2 3) and (2.4) we have solved the linearised perturbation equations governing the system. After applying the boundary conditions (7.1) at the perturbed boundary $r = 1+(\widetilde{\delta r}) \exp(i\omega t+im\theta)$, we get the following dispersion relation :

$$\omega^2 = (m-1)\{2m\lambda^2+\beta\}. \qquad \dots \quad (9.1)$$

This implies that the system is stable for all wave numbers $m > 1$. Further we note that the frequency of stable oscillations does not depend on the axial magnetic field prevailing inside the system Thus whatever be the functional form $f(r)$ of the axial magnetic field inside the system, the stability of the system against azimuthal disturbances is not influenced by the same (in the absence of surface currents).

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